13.4 Solving Radical Equations

What you should learn:

Goal 1 How to solve radical equations

Goal 2 How to use radical equations to solve real-life problems

Why you should learn it:

You can use radicals to solve many real-life problems, such as compensating for inaccuracy of a balance.

Goal 1 Solving a Radical Equation

Solving an equation that contains radicals is somewhat like solving a rational equation—you try to rewrite the equation as a polynomial equation. Then you solve the polynomial equation using the standard procedures. The following property plays a key role.

> **Squaring Both Sides of an Equation**
>
> If $a = b$, then $a^2 = b^2$.

Squaring both sides of an equation often introduces **extraneous solutions** of $a^2 = b^2$ that are *not* solutions of $a = b$. So when you use this procedure, it is critical that you check each solution in the *original* equation.

Before squaring both sides of an equation, you should isolate the radical expression on one side of the equation.

Example 1 *Solving a Radical Equation*

Solve $\sqrt{x} - 8 = 0$.

Solution

$$\begin{aligned} \sqrt{x} - 8 &= 0 && \textit{Rewrite original equation.} \\ \sqrt{x} &= 8 && \textit{Add 8 to both sides.} \\ (\sqrt{x})^2 &= 8^2 && \textit{Square both sides.} \\ x &= 64 && \textit{Simplify.} \end{aligned}$$

Check

$$\begin{aligned} \sqrt{x} - 8 &= 0 && \textit{Rewrite original equation.} \\ \sqrt{64} - 8 &\stackrel{?}{=} 0 && \textit{Substitute 64 for x.} \\ 8 - 8 &= 0 && \textit{Solution checks.} \end{aligned}$$

The equation has 64 as its solution. ■

Try solving the equation $\sqrt{x} + 8 = 0$. You will obtain the same solution, 64. For this equation, however, 64 is extraneous. Try to see why.

Example 2 Solving a Radical Equation

Solve $\sqrt{2x-1}+1=4$.

Solution

$$\begin{aligned}\sqrt{2x-1}+1&=4 && \textit{Rewrite original equation.}\\ \sqrt{2x-1}&=3 && \textit{Subtract 1 from both sides.}\\ (\sqrt{2x-1})^2&=3^2 && \textit{Square both sides.}\\ 2x-1&=9 && \textit{Simplify.}\\ 2x&=10 && \textit{Add 1 to both sides.}\\ x&=5 && \textit{Divide both sides by 2.}\end{aligned}$$

After checking $x = 5$ in the original equation, you can conclude that the equation has 5 as its solution. ■

Example 3 Solving a Radical Equation

Solve $x = \sqrt{x+2}$.

Solution

$$\begin{aligned}x&=\sqrt{x+2} && \textit{Rewrite original equation.}\\ x^2&=(\sqrt{x+2})^2 && \textit{Square both sides.}\\ x^2&=x+2 && \textit{Simplify.}\\ x^2-x-2&=0 && \textit{Write in standard form.}\\ (x-2)(x+1)&=0 && \textit{Factor.}\\ x&=2 \text{ or } x=-1 && \textit{Zero-Product Property}\end{aligned}$$

Try checking *each* x-value in the original equation. You will find that $x = 2$ checks, but $x = -1$ *does not* check. Thus, the equation has 2 as its solution. ■

Example 4 Finding the Geometric Mean of Two Numbers

The **geometric mean** of a and b is $\sqrt{ab}$. If the geometric mean of a and 4 is 12, what is a?

Solution

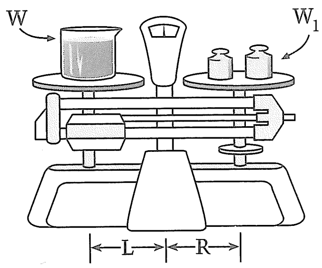
$$\begin{aligned}\text{Geometric mean}&=\sqrt{a\cdot 4}\\ 12&=\sqrt{4a}\\ 12^2&=(\sqrt{4a})^2\\ 144&=4a\\ 36&=a\end{aligned}$$

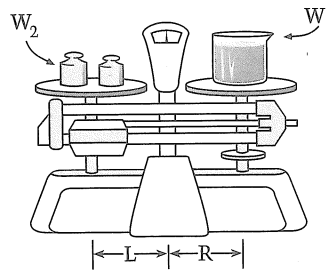
Check

$$\begin{aligned}12&\stackrel{?}{=}\sqrt{(36)(4)}\\ 12&\stackrel{?}{=}\sqrt{144}\\ 12&=12 \quad \textit{(checks)}\end{aligned}$$

The geometric mean of 36 and 4 is 12. ■

Goal 2 Solving Radical Equations in Real Life





Balance scales can give inaccurate results if the lengths of the left and right arms, L and R, are not *exactly* the same. Here is a technique that scientists use to be sure of an accurate weight. Let W be the true weight of an object.

1. Place the object on the left side of the scale. Counterbalance the right side with a weight of W_1. By a property of levers, you can write $\frac{W}{W_1} = \frac{R}{L}$.
2. Place the object on the right side of the scale. Counterbalance the left side with a weight of W_2: $\frac{W_2}{W} = \frac{R}{L}$
3. Equate the two expressions for $\frac{R}{L}$ and solve for W. The solution is $W = \sqrt{W_1 W_2}$. Thus, the true weight is the geometric mean of the weights obtained by weighing the object on the left and right sides of the scale.

Connections
Physical Science

Example 5 *Finding an Accurate Weight*

In science class, you weigh a sulfur sample only once—on the left side of a scale: It weighs 92 grams. Your teacher says the actual weight is 95 grams. If 95 grams is correct, what weight would you have obtained on the right side of the scale?

Solution The actual weight is $W = 95$ grams. On the left side of the scale, you observed the weight to be $W_1 = 92$ grams. Let W_2 be the weight that would be observed on the right side of the scale.

$W = \sqrt{W_1 W_2}$	*W is the geometric mean of W_1 and W_2.*
$95 = \sqrt{92 W_2}$	*Substitute 95 for W and 92 for W_1.*
$9025 = 92 W_2$	*Square both sides of the equation.*
$98.1 \approx W_2$	*Divide both sides by 92.*

You would have obtained 98.1 grams on the right side. ■

Communicating about ALGEBRA

Cooperative Learning

SHARING IDEAS about the Lesson

Work with a Partner and Use Your Own Words Solve the equations. Explain your steps.

A. $\sqrt{x} - 5 = 0$

B. $\sqrt{x} + 5 = 0$

C. $\sqrt{3x + 4} - 2 = 5$

D. $x - 2 = \sqrt{8 - x}$

EXERCISES

Guided Practice

CRITICAL THINKING about the Lesson

1. Describe a strategy for solving a radical equation. Then apply your strategy to solve $\sqrt{2x} - 10 = 0$.
2. Is $x = 25$ a solution of $\sqrt{x} = -5$? Why or why not?
3. One reason for checking a solution in the original equation is that you might have made an error in one of the steps of the solution. Describe another reason.
4. The geometric mean of 12 and x is 6. What is x?

Independent Practice

In Exercises 5–16, solve the equation. (Some of the equations have no solution.)

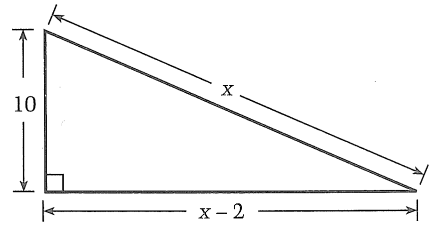
5. $\sqrt{x} - 10 = 0$
6. $\sqrt{x} - 1 = 0$
7. $\sqrt{-x} - \frac{1}{2} = \frac{3}{2}$
8. $\sqrt{3x} - 4 = 6$
9. $\sqrt{3x + 2} + 2 = 3$
10. $\sqrt{4 - x} - 5 = 1$
11. $4 = 6 - \sqrt{21x - 3}$
12. $10 = 17 + \sqrt{6x + 7}$
13. $\sqrt{\frac{1}{2}x - 5} - 1 = 11$
14. $\sqrt{\frac{1}{9}x + 1} - \frac{2}{3} = \frac{5}{3}$
15. $-5 - \sqrt{10x - 2} = 5$
16. $6 - \sqrt{7x - 9} = 3$

In Exercises 17–24, solve the equation.

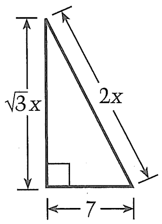
17. $x = \sqrt{20 - x}$
18. $x = \sqrt{6 - x}$
19. $\sqrt{-10x - 4} = 2x$
20. $x = \sqrt{100 - 15x}$
21. $\sqrt{77 - 4x} = x$
22. $2x = \sqrt{-13x - 10}$
23. $\frac{1}{2}x = \sqrt{x + 3}$
24. $x = \sqrt{4x + 32}$
25. Find the geometric mean of 8 and 32.
26. Find the geometric mean of 4 and 32.
27. The geometric mean of x and 5 is 15. What is x?
28. The geometric mean of 9 and x is 6. What is x?

Geometry **In Exercises 29 and 30, find the value of x.**

29.



30.

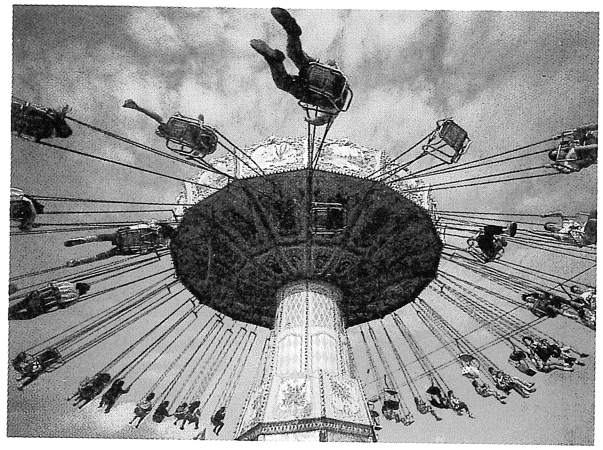


Spinning **In Exercises 31 and 32, use the following information.**

A ride at an amusement park spins in a circle of radius r, (in feet). The centrifugal force, F (in pounds), experienced by a passenger on the ride can be found by solving the equation

$$t = \sqrt{\frac{\pi^2 wr}{8F}}.$$

t is the number of seconds the ride takes to make one complete revolution and w is the weight, (in pounds), of the passenger.



31. A person who weighs 115 pounds is on a ride that is spinning at a rate of 10 seconds per revolution. The radius of the circular ride is 20 feet. How much centrifugal force does the person feel?

32. You are spinning on ice skates with your arms outstretched. As you pull your arms in toward your body, you are decreasing the radius, r. What effect does this have on t? What effect does it have on the skater? (Assume that the centrifugal force, F, is constant.)

The Speed of Sound **In Exercises 33 and 34, use the following information.**

The speed of sound near Earth's surface depends on the temperature. An equation that relates the speed, v (in meters per second), with the temperature, t (in degrees Celsius), is $v = 20\sqrt{t + 273}$.

33. Your friend is playing basketball 170 meters from you. You hear the sound of the ball hitting the backboard 0.5 seconds after seeing the ball hit the backboard. What is the temperature?

34. The temperature $-273°C$ is called absolute zero. What is the speed of sound at this temperature?

Rifle Velocity **In Exercises 35 and 36, use the following information.**

Because of the air resistance, the velocity of a bullet decreases after the bullet is fired. For a 25–06 rifle, the velocity, v (in feet per second), of the bullet after t seconds is given by $v = \frac{3200}{t + 1}$. If the rifle is held horizontally, the distance, h (in feet), the bullet *falls* is related to the time by the equation $t = \frac{1}{4}\sqrt{h}$.

35. A 25–06 rifle is fired horizontally at a target. The bullet hits the target in $\frac{1}{8}$ second. How fast is the bullet moving when it hits the target?

36. How far did the bullet fall before hitting the target?

The Winter Olympics Biathlon combines Nordic skiing with rifle shooting.